Florida VAM Methodology

Incorporated in Rule 6A-5.0411, Calculations of Student Learning Growth for Use in School Personnel Evaluations

Effective August 2015

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The Florida Value-Added Model (VAM)

Value-added analysis is a statistical method that estimates the effectiveness of a teacher by seeking to isolate the contribution of the teacher to student learning. Conceptually, a portion of the difference between a student's actual score on an assessment and the score they were expected to achieve is the estimated "value" that the teacher added during the year to the teacher's students' learning growth with respect to the content tested. A student's expected score is based on the student's prior test score history and measured characteristics, as well as how other students in the state actually performed on the assessment.

The value-added models implemented for the State of Florida are covariate adjustment models that include up to two prior assessment scores (except in the grade 4 statewide, standardized assessment models, where only one prior assessment is available) and a set of measured characteristics for students. The models use error-in-variables regression to account for the measurement error in the covariates used.

Executive Summary

This document contains a technical description of the data sources, formula, covariates, and methodology for calculating VAM scores. It should be read in conjunction with the main rule text provided in Rule 6A-5.0411, F.A.C. It uses the definitions provided in the main rule text. Section 1: Data, describes the processes districts use to submit data to the Department for use in VAM calculations. It requires the Department to establish a schedule by which the Department will extract and process data related to students, courses, and teachers, and includes opportunities for districts to revise and correct information provided to the Department. It also includes a list and description of the variables, referred to as covariates, controlled for in the model to help ensure that the diversity of students is taken into account when assigning scores to teachers using the model. These include, for example, student disabilities and the percentage of days a student was in attendance. Section 2 describes the statistical models for VAM, including the differences between the models used for ELA and mathematics and the model used for Algebra I. It includes a description of how the model takes measurement error in test scores for prior assessments into account. It also describes the processes used to derive fixed and random effects model coefficients used to generate student expected scores and how model converge issues are resolved. Section 3 describes the process used to create final value added model scores for teachers by grade and subject. Section 4 describes how value added model scores for different grades, subjects and years are aggregated into final scores across multiple grades, subjects and years. It also describes the files and data elements delivered to districts containing information produced by the value added model. Appendix A describes the processes and data elements used to create the reference files.

Section 1: Data

To measure student growth and to attribute that growth to educators, at least two sources of data are required: student assessment scores that can be observed over time and information describing how students are linked to schools, teachers, and courses (i.e., identifying which teachers teach which students for which tested subjects and which school[s] those students attended). In addition, Florida's value-added models also use other information, such as student characteristics and attendance data. The following sections describe the data used for model estimation in more detail.

Assessment Scores

Florida's value-added models draw on data from statewide assessment programs in Grades 3–10 in ELA, Grades 3-8 in Mathematics and on Algebra 1 End-of-Course scores. Models are estimated separately by grade and subject using scores from each grade/subject (e.g., Grade 5 mathematics) as the outcome, with additional covariates as described in the "Other Covariates" section.

<u>Up to two prior years of achievement scores are included for each student</u>. This covariate is used to control for effects related to a student's prior test scores with content aligned to the statewide standardized assessment (English/Language Arts or Mathematics) being modeled. The variables used are the developmental scale score on the prior year subject-relevant assessment and, when available, the developmental scale score on the subject-relevant assessment from two years prior. When the subject-relevant developmental scale score from two years prior is not available, a dichotomous missing value indicator variable is used.

Student/Teacher/Course Data

Course enrollment data used in the VAM calculations are drawn from the Student/Teacher/Course File, which is compiled through the following process:

1. Survey 2. School districts submit Survey 2 data to the Department's Student Information System and Staff Information System, pursuant to Rule 6A-1.0014, F.A.C. (Comprehensive Management Information System) and Rule 6A-1.0451, F.A.C. (Florida Education Finance Program Student Membership Surveys).

2. Roster Verification for Survey 2. School districts shall verify Survey 2 class rosters for use in the VAM calculations, in accordance with s. 1012.34, F.S. Districts may choose to use the Department's roster verification tool for this purpose. Districts utilizing the Department's roster verification tool must submit Survey 2 data by the first Survey 2 draw down date established by the Department. After the first Survey 2 draw down date, the Department will populate the roster verification tool with course enrollment data submitted by school districts in Survey 2.

   Districts that do not use the Department's roster verification tool, but use an independent District roster verification process must submit verified course enrollment data in Survey 2.

   Districts shall have at least four weeks from the date the roster verification tool is opened to verify the roster data.

3. Survey 3. School districts submit Survey 3 data to the Department's Student Information System and Staff Information System, pursuant to Rule 6A-1.0014, F.A.C. (Comprehensive Management Information System) and Rule 6A-1.0451, F.A.C. (Florida Education Finance Program Student Membership Surveys).

4. Roster Verification for Survey 3. School districts shall verify Survey 3 class rosters for use in the VAM calculations, in accordance with s. 1012.34, F.S. Districts may choose to use the Department's roster verification tool for this purpose. Districts utilizing the Department's roster verification tool must submit Survey 3 data by the first Survey 3 draw down date established by the Department. After the first Survey 3 draw down date, the Department will populate the roster verification tool with course enrollment data submitted by school districts in Survey 3.
   Districts that do not use the Department's roster verification tool, but use an independent District roster verification process must submit verified course enrollment data in Survey 3.

   Districts shall have at least four weeks from the date the roster verification data tool is opened to verify the roster data.

5. Survey 2 and 3 Match. Districts are permitted to make local decisions to exclude students from a teacher's VAM calculation if he or she changed schools or left the district between survey periods. After the final draw down dates established for Surveys 2 and 3 established by the Department, the Department matches students across both surveys and identifies if each student was present at the same district or at the same school, and provides the results of these matches back to districts. Districts shall make final corrections to the student/teacher/course files and submit back to the Department final files for both surveys for the inclusion in the VAM analysis. Districts shall have at least two weeks from the date the Survey 2 and 3 match files are provided by the Department to make final corrections.

6. Final Student/Teacher/Course Files. The final student/teacher/course files are compiled from the Survey 2 and 3 Match, if the District opts for such a match, or if not, from the post-verification roster data described above.

Teachers and students associated with the courses shown in the document "Florida VAM Course List" are included in analyses. See the section on "Construction of Z matrix" for more information on how student/teacher/course data are used in models.

Other Covariates

Both student and classroom characteristics are statistically controlled for in Florida's value-added models. Using these characteristics in the value-added model is intended to help ensure fair comparisons across teachers of diverse groups of students.

Following is a list of factors (beyond prior assessment scores described earlier) included in each model and a description of how they are derived.

<u>The number of subject-relevant courses in which the student is enrolled</u>: The number of subject-relevant courses in which a student is enrolled. Relevant courses are listed in the document "Florida VAM Course List". This covariate is used to control for the effects related to the amount of instruction in the subject the student received during the year. It counts, for each student, the number of courses in which he or she was enrolled during either Survey 2 or Survey 3 of the most recent school year that are associated with the statewide standardized assessment being modeled (English/Language Arts or Mathematics). The variables that make up this covariate are three binary variables indicating whether the student was enrolled in 2 or more, 3 or more, and 4 or more subject-relevant courses as reported by school districts via the course number and period data elements contained within the student course and teacher course reporting formats of the student information and staff information systems.

<u>A student's disabilities</u>: This covariate is used to control for effects related to which disabilities, if any, the student has. It is measured using an array of ten binary variables, each indicating the presence or absence of a specific exceptionality as reported by school districts via the primary exceptionality and/or other exceptionality data elements contained within the exceptional student reporting format of the student information system during survey 2 or 3. The exceptionalities used within the model are limited to language impaired; deaf or hard of hearing; visually impaired; emotional/behavioral disabilities; specific learning disability; dual sensory impaired; autistic; traumatic brain injured; other health impaired; and other intellectual disability.

<u>A student's English Language Learner (ELL) status</u>: This covariate is used to control for effects related to whether a student has limited English proficiency. It is based on whether a student has been identified as an ELL and is enrolled in a

program or receiving services that are specifically designed to meet the instructional needs of ELL students. These data are reported by school districts via the ELL indicator in the student demographic format, and the English Language Learners entry date contained within the English language learners format, of the student information system. It is measured using four binary variables indicating whether the student has been an ELL for less than two years; at least two years but less than 4 years; at least 4 years but less than 6 years; or at least 6 years or longer.

<u>Gifted status</u>: This covariate is used to control for effects related to whether or not the student is gifted. It is measured using a binary variable, indicating the presence or absence of a the gifted exceptionality as reported by school districts via the primary exceptionality and other exceptionality data elements contained within the exceptional student reporting format of the student information system during Survey 2 or 3.

<u>Student attendance</u>: This covariate is used to control for effects related to student attendance. It is measured by a continuous variable that indicates the percentage of days a student was enrolled that the student was in attendance during the school year as reported by school districts in the student information system via the days present and days absent data elements from the prior school status/attendance format of the student information system during Survey 2 and 3. The variable is computed as the ratio of the sum of the days present to the sum of the days present plus days absent across all schools and surveys for the year for the student.

<u>Student mobility</u>: This covariate is used to control for effects related to changing schools during the school year. It is measured by a continuous variable that is a count of the number of schools beyond the first one that reported the student via the count of the unique combinations of district, school, student, entry date and withdrawal dates, as reported via the prior school and student attendance format during Survey 2 and 3.

<u>Difference from modal age in grade</u>: This covariate is used to control for effects related to differences in a student's age from the most common age for students enrolled in the same grade across the state and is included as an indicator of retention or acceleration. It is measured by a continuous variable that computes the difference, in years, between the student's age on September 1 of the school year and the modal age of all students in the same grade as reported by school districts in the student information system via the date of birth data element from the student demographic format of the student information system during Survey 2 or 3.

<u>Class size</u>: This covariate is used to control for effects related to the number of students in a class. It is measured by a group of up to 6 continuous variables representing the subject-relevant courses in which the student is enrolled. Each variable represents a different class and is the sum of students enrolled in the same class as reported by school districts in the student information system via the course number and period data elements contained within the student course and teacher course reporting formats of the student information and staff information systems during Survey 2 and 3.

<u>Homogeneity of students' entering test scores in the class</u>: This covariate is used to control for the variation in student proficiency within a classroom at the beginning of the year. It is measured by a group of continuous variables that represent each of up to 6 subject-relevant classes in which the student is enrolled. Each of these variables computes the difference between developmental scale scores located at the 25th percentile and 75th percentile of students assigned to the teacher who are enrolled in the same class on the prior year's assessment. When the student is enrolled in fewer than 6 subject-relevant classes, a binary missing value indicator variable is used for each class beyond the first one for which there is no data for the student.

In addition to the variables listed above, the Algebra I EOC models include the following covariates:

Mean prior test score: This covariate is used to control for the effect of the overall incoming proficiency level of students in the class. It is measured by a group of continuous variables that represent each of up to 6 subject-relevant classes the student is enrolled in. For each of these classes, it is the average of the most recent prior score on the statewide, standardized assessment in Mathematics for all students within the class. When the student is enrolled in fewer than 6 subject-relevant classes, a binary missing value indicator variable is used for each class beyond the first one for which there is no data for the student.

Percent gifted: This covariate is used to control for the effect of the overall proportion of the class that is gifted. It is measured by a group of continuous variables that represent each of up to 6 subject-relevant classes the student is enrolled in. For each of these classes, it is the percentage of students in the class identified as gifted as reported by school districts via the primary exceptionality and/or other exceptionality data elements contained within the exceptional student reporting format of the student information system during Survey 2 or 3. When the student is enrolled in fewer than 6 subject-relevant classes, a binary missing value indicator variable is used for each class beyond the first one for which there is no data for the student.

Percent at modal age in grade: This covariate is used to control for the effect of differences in age from the most common age for students in the grade among the students enrolled in the class as an indicator of what proportion of students in the class may be accelerated or retained students. It is measured by a group of continuous variables that represent each of up to 6 subject-relevant classes the student is enrolled in. For each of these classes it is the percentage of students whose age on September 1 of the school year is the same as the modal age of all students in the same grade as reported by school districts in the student information system via the date of birth data element from the student demographic format of the student information system during Survey 2 or 3. When the student is enrolled in fewer than 6 subject-relevant classes, a binary missing value indicator variable is used for each class beyond the first one for which there is no data for the student.

Reference File

In preparation for the analysis, a single student-level data file, called the Reference File, is created for each grade/subject combination (i.e., 6th grade ELA, 6th grade Mathematics). These files are compiled from the following data sources:

- Relevant course lists (Located in the document "Florida VAM Course List")
- Student/Teacher/Course File
- Surveys 2 and 3 (extracted from survey data after the final Survey 2/3 down dates):
  - Student demographic information
  - Student attendance information
  - Student ELL status information
  - Student exceptionality information
- Assessment data
- Staff demographics
- School Information

Within the model year, the relevant course list is used for subsetting both the Teacher Course File and the Student Course File for the relevant courses for each subject. Summer courses are excluded. The Student Course Files are subset for the relevant courses, and then unique records according to Student_Unique_ID, Year, Survey, District_ID, School_ID, Period, Section_Number, Course_Number are kept in the file. The Teacher Course File is subset for the relevant courses, and then unique records according to Teacher_ID, Year, Survey, District_ID, School_ID, Period, Section_Number, Course_Number are kept in the file. Then the Teacher Course File and the Student Course Files are joined by Year, Survey, District, School, Period, Section_Number, Course_Number.

The student demographic file is sorted by Year, Student_Unique_ID, Survey, District_enroll, School_enroll, District_ID, gender, birthyear, and the last record within each year for each student is retained. The attendance records are sorted by Student_Unique_ID, District ID, School ID. The days present and days absent across all records for a student are accumulated across districts and schools, and the last student record is retained. ELL information is sorted by Student_Unique_ID, survey, and the last record is retained. Student exceptionality data are sorted by Student_Unique_ID, Survey, district, school. Exceptionality flags are accumulated across all surveys, districts, and schools for the student, and the last record is retained. These four files (demographic, attendance, ELL, and exceptionality) are then merged by Student_Unique_ID.

Up to four years of student assessment data are merged together by Student_Unique_ID. Merges are verified using the Levenshtein distance test based on first and last names. Data are then reshaped from a long dataset (one observation per student/teacher/course/period) to a wide dataset (one observation per student). Assessment data are then merged to student data by Student_Unique_ID. Merges are again verified using the Levenshtein test. When a student has no current assessment information or the student has no prior assessment data in the most recent year used among the covariates (immediate prior year for ELA and Mathematics, but can be up to 2 years earlier than the current year for Algebra) then the student is removed from the analysis.

Classes are defined as the unique combination of variables district/school/teacher/course/period and reported in the data as ClassID. For each student/subject/year, N unique ClassID combinations are retained for the analysis. Analyses are limited to a maximum of 6 subject relevant courses.

Appendix A contains a list of the variables, source files, and code values used to generate the reference file.

During the creation of the files to be used in model analysis, some of the data may be excluded. Table 1 lists the rules for excluding assessment information from the calculations.

Table 1. Exclusions and deletions

Exclusion code	Exclusion description	Exclusion implementation (describe variables and values that lead to exclusion)
E.1	All grade 3 student records in the current year will be eliminated	
E.2	If a student has multiple records in a single year with contradictory grade levels, all records will be eliminated	
E.3	If a student has multiple statewide, standardized assessment records in a single year or, for EOCs within the same reporting period (i.e. Winter or Spring), with contradictory test scores in the same subject, all records for that subject are rejected for	

	the purposes of the analysis. However, for EOCs within the same year but different periods, the best score is retained and the others excluded.	
E.4	Student records with missing, or invalidated test scores are removed	
E.5	All grade 9 and 10 student records for the Mathematics statewide, standardized assessment for all years will be eliminated	
E.6	In the ELA and Mathematics models, exclude students without both current and immediate prior year assessment data for the state standard assessments.	Eliminate records missing the prior year test score. If ScaleScore_<yy-1> missing then delete record from analysis.
E.7	If a student's current tested grade is lower than the student's prior tested grade, eliminate the record	If tested grade in the current year is less than the tested grade in the prior year, exclude the record from analysis. If Testedgrade yy < Testedgrade yy-1, exclude the record. If Testedgrade yy-1 < Testedgrade yy-2, exclude the record
E.8	For EOCs, students must have at least one immediate prior grade.	Students in grade 8 must have 1st or 2nd prior year 7th grade (M_1_7 or M_2_7). Students in grade 9 must have 1st or 2nd prior year 8th grade (M_1_8 or M_2_8).
E.9	Prior attendance records from survey 3 with "Y" or "S" codes are not used for the record	These students are summer term students and should not be in the analysis

Timeline for Data Submission and Extraction

No later than September 30 of each year, the Department will post the timeline of data submission and extraction activities associated with the VAM calculation on its website at (http://www.fldoe.org/teaching/performance-evaluation) and shall notify districts of these dates. The timeline shall provide a schedule for the following activities:

Survey 2 first draw down date

Complete Roster Verification for Survey 2 (for Districts using the Department's roster verification tool)

Complete Roster Verification for Survey 2 (for Districts not using the Department's roster verification tool)

Survey 3 first draw down date

Complete Roster Verification for Survey 3 (for Districts using the Department's roster verification tool)

Complete Roster Verification for Survey 3 (for Districts not using the Department's roster verification tool)

Final Survey 2/3 draw down date

District submission of Survey 2/3 Match Requests

District final corrections of Survey 2/3 match data

Section 2: Description of the Statistical Model and Its Implementation

This section provides the technical description of Florida's value-added models and their computational implementation.

Covariate Adjustment Model

The statistical value-added model implemented for the State of Florida is a covariate adjustment model, as the current year observed score is conditioned on prior levels of student achievement as well as other covariates that may be related to the student or classroom characteristics.

Models are run separately by grade, subject, and year. In its most general form, the model can be represented as follows:

$$\mathrm{y}_{ti} = \mathbf{X}_i\boldsymbol{\beta} + \mathrm{y}_{t-1,i}\gamma_1 + \mathrm{y}_{t-2,i}\gamma_2 + \mathbf{Z}_{1i}\boldsymbol{\theta}_1 + \mathbf{Z}_{2i}\boldsymbol{\theta}_2 + e_{ti}$$

where the terms in the model are defined as follows:

- y_{ti} is the observed score at time *t* for student *i.*
- $\mathbf{X}_i$ is the matrix for the student and classroom demographic variables for student *i.*
- $\boldsymbol{\beta}$ is a vector of coefficients capturing the effect of any covariates included in the model except prior test score.
- $y_{t-r,i}$ is the prior test score at time *t-r* ($r \in \{1,2\}$).
- γ_1 is the coefficient capturing the effects of the most recent prior test score.
- γ_2 is the coefficient capturing the effects of the second prior test score. Elsewhere in this document, γ_1 and γ_2 are concatenated such that $\boldsymbol{\gamma}' = \{\gamma_1, \gamma_2\}$ is the coefficient vector capturing the effects of up to two prior test scores.
- $\mathbf{Z}_{1i}$ is a design matrix with one column for each teacher and one row for each student record in the data file. The entries in the matrix indicate the association between the student record represented in the row and the teacher represented in the column.
- $\boldsymbol{\theta}_1$ is the vector of teacher random effects.
- $\mathbf{Z}_{2i}$ is a design matrix with one column for each school and one row for each student record in the data file. The entries in the matrix indicate the association between the student record represented in the row and the school represented in the column. Elsewhere in this document, $\mathbf{Z}_1$and $\mathbf{Z}_2$ are concatenated such that $\mathbf{Z} = \{\mathbf{Z}_1, \mathbf{Z}_2\}$.
- $\boldsymbol{\theta}_2$ is the vector of school random effects. Corresponding to $\mathbf{Z} = \{\mathbf{Z}_1, \mathbf{Z}_2\}$, define $\boldsymbol{\theta}' = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2')$.
- e_{ti} captures all residual student-level factors contributing to student achievement.

Because Florida's VAM model treats these vectors of effects as random and independent from each other, it is assumed that the distributions of teacher and school effects are approximately normal about a mean of 0 ($\boldsymbol{\theta}_q \sim N(0, \sigma^2_{\boldsymbol{\theta}_q})$) for each level of *q* where $q \in \{1,2\}$, with 1 referencing teacher and 2 referencing school. In the subsequent sections, the notation $\boldsymbol{\delta}' = \{\boldsymbol{\beta}', \boldsymbol{\gamma}'\}$ is used to refer to the covariate coefficient vectors collectively, and $\mathbf{W} = \{\mathbf{X}, \mathbf{y}_{\mathrm{t}-1}, \mathbf{y}_{\mathrm{t}-2}\}$ is used to refer to the covariate values collectively in order to simplify computation and explanation.

The statistical model applied to the statewide, standardized assessment and EOC data decomposes total variation in achievement into three orthogonal components: variance between schools (the school component), variance between teachers (the teacher component), variance among residual student-level factors.

While all parameters are estimated simultaneously, conceptually it is helpful to consider the levels separately. First, student-level prior assessment data and other covariates are used to establish a statewide conditional expectation, called an expected score. The expected score is based on the student's prior test score history and measured characteristics, as well as how other students in the state actually performed on the assessment.

However, schools exhibit differential amounts of growth. The school component refers to how much higher or lower the school's students scored, on average, compared to other students in the same grade and subject in the state after adjusting for the covariates. Similarly, the teacher component refers to how much higher or lower the teacher's students scored, on average, compared to other students within the school after adjusting for the covariates.

If the school component were *excluded* and the model included only the teacher component, then legitimate differences between teachers could be exaggerated, as the model would ignore any variation in effectiveness among schools. In other words, some teachers could appear to have higher (or lower) value-added scores because their estimated scores would include things common to all students in a school and may not be due to teachers, such as principal leadership.

In contrast, when the school component is *included*, then some of the legitimate differences between teachers could be minimized. For example, if all teachers at a school are highly effective, the school component would capture this common effectiveness and attributed it to the school, rather than the individual teachers.

As a result, when estimating a value-added model, it needs to be determined whether the model should:

- estimate the common school component, thus potentially removing some legitimate differences between teachers;
- ignore the common school component and assume that any difference in learning across classes is entirely a function of classroom instruction; or
- estimate the common school component, and then attribute some portion of this back to teachers—i.e., find some middle ground where teacher value-added scores include some but not all of the common school component.

Adding none of the school component (0%) to teachers' value-added scores essentially creates a model with different growth expectations for otherwise similar students who attend different schools. A teacher whose students exhibit high growth in a school where high growth is typical could earn a lower value-added score than a second teacher whose students exhibit less growth than the first teacher, but who teaches at a school where lower growth is typical. In contrast, adding the entire school component (100%) to teachers' value-added scores creates a model with the same statistical expectations for student outcomes, regardless of the school the student attends. Teachers with high student growth in high-growth schools will earn higher value-added scores than teachers with lower growth at low-growth schools, regardless of how each teacher performed relative to the other teachers at the school.

The Student Growth Implementation Committee (SGIC), the statewide committee tasked with providing direction on value-added model implementation, recommended that 50 percent of the school component should be added to the teacher component. Teacher value-added scores from the statewide, standardized assessment models result from the following calculation:

Teacher Value-Added Score = Unique Teacher Component + .50 * Common School Component

This formula recognizes that some of the school component is a result of teacher actions within their schools, and therefore teachers should receive some credit for the typical growth of students in their school in their overall value-added scores.

For the Algebra I EOC models, the SGIC determined that none of the school component should be attributed back to teachers. The SGIC made this decision because more than one-third of schools have only one or two Algebra I teachers teaching grade 9 students, and more than half of schools have only one or two Algebra I teachers teaching grade 8 students. In these situations, it is difficult to distinguish between teacher effects and the common school component, and so the SGIC decided that attributing the school component back to the teacher effect was unnecessary.

Accounting for Measurement Error in the Prior Scores

Florida's value-added models account for measurement error in test scores through an error-in-variables approach. Accounting for measurement error is important because otherwise bias will still remain, even if multiple scores are used.

To describe how the model accounts for measurement error, we stack the rows for all students in a grade, subject, and year, drop the *i* subscript from the model described previously, and re-express the true score regression equation as follows:

$$\mathbf{y}_{\mathrm{t}}^{*} = \mathbf{X}\boldsymbol{\beta} + \mathbf{y}_{t-1}^{*}\boldsymbol{\gamma}_1 + \mathbf{y}_{t-2}^{*}\boldsymbol{\gamma}_2 + \mathbf{Z}\boldsymbol{\theta} + e_t$$

We use * to denote the variables without measurement error. For convenience, define the matrices $\mathbf{W} = \{\mathbf{X}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}\}$, $\mathbf{W}^* = \{\mathbf{X}, \mathbf{y}_{t-1}^*, \mathbf{y}_{t-2}^*\}$, and $\boldsymbol{\delta}' = \{\boldsymbol{\beta}', \boldsymbol{\gamma}'\}$. Let N be the number of students included in the model and $p_{\boldsymbol{X}}$ be the number of columns in $\mathbf{X}$, so that $\mathbf{W}^*$ has N rows and $p = p_{\boldsymbol{X}} + 2$ columns.

Label the matrix of measurement errors associated with $\mathbf{y}_{t-1}, \mathbf{y}_{t-2}$ by $\boldsymbol{U}_{\mathbf{2}}$. Define $\boldsymbol{U} = [\mathbf{0}_{\boldsymbol{p}_{\mathbf{X}}} \quad \boldsymbol{U}_2]$, where $\mathbf{0}_{\boldsymbol{p}_{\mathbf{X}}}$ is an $N \times p_{\mathbf{X}}$ matrix with elements of 0, so that $\boldsymbol{U}$ has the same dimension as $\mathbf{W}$, but only the final 2 columns of $\boldsymbol{U}$ are non-zero, so $\mathbf{W} = \mathbf{W}^* + \mathbf{U}$. If those measurement errors were observed, the parameters $\{\boldsymbol{\delta}', \boldsymbol{\theta}'\}$ can be estimated by solving the following mixed model equations:

$$\begin{pmatrix} \boldsymbol{W}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{W}^* & \boldsymbol{W}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \boldsymbol{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W}^* & \boldsymbol{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{\delta} \\ \boldsymbol{\theta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{W}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{\mathrm{t}}^* \\ \boldsymbol{Z}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{\mathrm{t}}^* \end{pmatrix}$$

Let $\boldsymbol{\Omega}$ be a diagonal matrix of dimension *N* with diagonal elements $\sigma_{ti}^2 = \sigma_e^2 + \delta_1^2\sigma_{u,1,i}^2 + \delta_2^2\sigma_{u,2,i}^2$, where $\sigma_{u,1,i}^2$ and $\sigma_{u,2,i}^2$ are the known measurement variance of the 2 prior test scores and δ_1 and δ_2 are the coefficients on those prior scores. The matrix $\mathbf{D}$ is made up of *2* diagonal blocks, one for teachers and one for schools. Each diagonal is constructed as $\sigma_{\mathrm{q}}^2\mathbf{I}_q$ where $\mathbf{I}_q$ is an identity matrix with dimension equal to the number of units at level *q*, and σ_q^2 is the estimated variance of the random effects among units at level *q*. When concatenated diagonally, the square matrix $\mathbf{D}$ has dimension $m = J_1 + J_2$, where J_q is the number of units at level *q*.

Two complications intervene. First, we cannot observe $\mathbf{U}$, and second, the unobservable nature of this term, along with the heterogeneous measurement variance in the dependent variable, renders this estimator inefficient.

Addressing the first issue, upon expansion we see that:

$$\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} = (\mathbf{W}^{*\prime} + \mathbf{U}')\boldsymbol{\Omega}^{-1}(\mathbf{W}^* + \mathbf{U}) = \mathbf{W}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{W}^* + \mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{W}^* + \mathbf{W}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{U} + \mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U}$$

Taking expectation over the measurement error distributions and treating the true score matrix, $\mathbf{W}^*$, as fixed , we have

$$\mathrm{E}(\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}) = \mathrm{E}\left((\mathbf{W}^{*\prime} + \mathbf{U}')\boldsymbol{\Omega}^{-1}(\mathbf{W}^{*} + \mathbf{U})\right) = \mathbf{W}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{W}^{*} + \mathrm{E}(\mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U})$$

And then rearranging terms gives

$$\mathbf{W}^{*\prime}\boldsymbol{\Omega}^{-1}\mathbf{W}^{*} = \mathrm{E}(\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}) - \mathrm{E}(\mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U})$$

We also have $\mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W}^{*} = \mathrm{E}(\mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W})$ with the expectation taken over the measurement error distributions associated with observed $\mathbf{W}$, and $\begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{\mathrm{t}}^{*}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{\mathrm{t}}^{*}\end{pmatrix} = \mathrm{E}\begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\end{pmatrix}$ with the expectation taken over the measurement error distributions associated with observed $\mathbf{y}_{t}$.

As described earlier, $\boldsymbol{\Omega}$ is a diagonal matrix of dimension *N* with diagonal elements $\sigma_{ti}^2 = \sigma_e^2 + \delta_1^2\sigma_{u,1,i}^2 + \delta_2^2\sigma_{u,2,i}^2$, where $\sigma_{u,1,i}^2$ and $\sigma_{u,2,i}^2$ are the known measurement variance of the 2 prior test scores and δ_1 and δ_2 are the coefficients on those prior scores. Because the measurement error of the prior score varies depending on the value of the prior score, $\sigma_{u,1,i}^2$ and $\sigma_{u,2,i}^2$ vary across students. With the above, we can define the mixed model equations as:

$$\begin{pmatrix}\mathrm{E}(\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}) - \mathrm{E}(\mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U}) & \mathrm{E}(\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z})\\ \mathrm{E}(\mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W}) & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1}\end{pmatrix}\begin{pmatrix}\boldsymbol{\delta}\\ \boldsymbol{\theta}\end{pmatrix} = \mathrm{E}\begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\end{pmatrix}$$

Using observed scores and measurement error variance, the mixed model equations are redefined as:

$$\begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} - \mathrm{E}(\mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U}) & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1}\end{pmatrix}\begin{pmatrix}\boldsymbol{\delta}\\ \boldsymbol{\theta}\end{pmatrix} = \begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\end{pmatrix}$$

Observed Values for $\mathrm{E}(\mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U})$

As indicated, $\mathbf{U}$ is unobserved, and so solving the mixed model equation cannot be computed unless $\mathbf{U}$ is replaced with some observed values. First, the mixed model equations are redefined as:

$$\begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} - \mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1}\end{pmatrix}\begin{pmatrix}\boldsymbol{\delta}\\ \boldsymbol{\theta}\end{pmatrix} = \begin{pmatrix}\mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{y}_{t}\end{pmatrix}$$

where $\mathbf{S}$ now stands in place of $\mathrm{E}(\mathbf{U}'\boldsymbol{\Omega}^{-1}\mathbf{U})$ and is subsequently defined as a diagonal "correction" matrix with dimensions $p \times p$ accounting for measurement variance in the predictor variables

Recall that we previously defined $\boldsymbol{\Omega}$ as $\mathrm{diag}(\sigma_{t1}^2, \sigma_{t2}^2, \ldots, \sigma_{tN}^2)$ and the matrix of unobserved disturbances is:

$$\boldsymbol{U} = [\mathbf{0}_{p_{\mathrm{X}}} \quad \boldsymbol{U}_2]$$

where $\mathbf{0}_{p_{\mathrm{X}}}$ is a $N \times p_{\mathbf{X}}$ matrix with elements of 0, and:

$$\boldsymbol{U}_2 = \begin{bmatrix}u_{11} & u_{12}\\ u_{21} & u_{22}\\ \vdots & \vdots\\ u_{N1} & u_{N2}\end{bmatrix}$$

The theoretical result of the matrix operation yields the following symmetric matrix:

$$\mathbf{U}_2{}'\boldsymbol{\Omega}^{-1}\mathbf{U}_2 = \begin{bmatrix} \sum_{i=1}^{N} \frac{1}{\sigma_{ti}^2} u_{i1}^2 & \sum_{i=1}^{N} \frac{1}{\sigma_{ti}^2} u_{i2} u_{i1} \\ \sum_{i=1}^{N} \frac{1}{\sigma_{ti}^2} u_{i1} u_{i2} & \sum_{i=1}^{N} \frac{1}{\sigma_{ti}^2} u_{i2}^2 \end{bmatrix}$$

The theoretical result is limited only because we do not observe u_{i1} and u_{i2}, since they are latent. However, $E(u_{i1}u_{i1}) = \sigma_{i1}^2$ and $E(u_{i2}u_{i2}) = \sigma_{i2}^2$, where σ_{i1}^2 and σ_{i2}^2 are taken as the conditional standard errors of measurement, which depend on the value of the prior scores, for student *i*. The theoretical result also simplifies because variances of measurement on different variables are by expectation uncorrelated, i.e. $E(u_{i1}u_{i2}) = E(u_{i2}u_{i1}) = 0$.

Because the conditional standard error of measurement varies for each student *i* and the off-diagonals can be ignored, let the $p \times p$ matrix $\mathbf{S}$ be:

$$\mathbf{S} = \mathrm{diag}\left(0, \ldots 0, \sum_{i=1}^{N} \frac{1}{\sigma_{ti}^2} \sigma_{u,1,i}^2, \sum_{i=1}^{N} \frac{1}{\sigma_{ti}^2} \sigma_{u,2,i,}^2\right)$$

where $\sigma_{\mathrm{ti}}^2 = \sigma_{\mathrm{e}}^2 + \delta_1^2 \sigma_{u,1,i}^2 + \delta_2^2 \sigma_{u,2,i}^2$, $\sigma_{u,1,i}^2$ denotes the measurement variance for the first prior score, and $\sigma_{u,2,i}^2$ denotes the measurement variance for the second prior score.

Computing the Value-Added Model

The implementation of the value-added model uses the Expectation-Maximization (EM) algorithm to solve the mixed model equations. The solutions for the fixed effects and predictions for the random effects are obtained using the Expectation-Maximization (EM) algorithm via the following steps:

1. Construct starting values for the variances of the random effects including σ_e^2 and σ_q^2 for all levels of *q*. These are used in the matrices $\boldsymbol{\Omega}$ and **D**, respectively. Typical starting values are 10,000; 1,000; and 1,000 for residual, teacher, and school variance respectively.
2. Solve the linear system for $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$.
3. Update the values of the variances of the random effects including σ_e^2 and σ_q^2 using the methods described above.
4. Iterate between steps 2 and 3 until $\left|\boldsymbol{\delta}_p^{\eta} - \boldsymbol{\delta}_p^{\eta-1}\right| < 1\text{e-}5\ \forall\ p$, where η indexes the iterations.

Standard Errors of Fixed and Random Effects

The standard errors of the fixed and random effects can be computed as:

$$\mathrm{Var}\begin{pmatrix}\boldsymbol{\delta}\\ \boldsymbol{\theta}\end{pmatrix} = \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} - \mathbf{S} & \mathbf{W}'\,\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\,\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{W}'\,\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\,\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} - \mathbf{S} & \mathbf{W}'\,\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\,\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix}^{-1}$$

Note that

$$\begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} - \mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W} - \mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{pmatrix}^{-1}$$

$$=\begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix}^{-1} \left[\begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix} + \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\right] \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix}^{-1} + \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix}^{-1}$$

Let $\begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{C} \end{pmatrix}$ and $\begin{pmatrix} \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{W}-\mathbf{S} & \mathbf{W}'\boldsymbol{\Omega}^{-1}\mathbf{Z} \\ \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{W} & \mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{12}' & \mathbf{C}_{22} \end{pmatrix}$. Then $\mathbf{C}_{11} = (\mathbf{A}-\mathbf{B}\mathbf{C}^{-1}\mathbf{B}')^{-1}$, $\mathbf{C}_{12} = -(\mathbf{A}-\mathbf{B}\mathbf{C}^{-1}\mathbf{B}')^{-1}\mathbf{B}\mathbf{C}^{-1}$ and

$$\mathbf{C}_{22} = \mathbf{C}^{-1} + \mathbf{C}^{-1}\mathbf{B}'(\mathbf{A}-\mathbf{B}\mathbf{C}^{-1}\mathbf{B}')^{-1}\mathbf{B}\mathbf{C}^{-1}.$$

In order to compute the standard errors of the random effects, it is assumed that teachers teach in only one district, that students do not move across districts within a school year, and that $\mathbf{C}_{22}$ therefore is block diagonal. Under this assumption $\mathbf{C}^{-1}$ can be computed efficiently and the other computations also become tractable even for very large data sets. If there are some students who were in two or more districts during the current year, a few entries in the matrix will not be in the block diagonal, but these will simply be ignored for the purposes of computing the variance terms.

Now

$$var\begin{pmatrix} \boldsymbol{\delta} \\ \boldsymbol{\theta} \end{pmatrix} = \begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{12}' & \mathbf{C}_{22} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{11}\mathbf{S}\mathbf{C}_{11} & \mathbf{C}_{11}\mathbf{S}\mathbf{C}_{12} \\ \mathbf{C}_{12}'\mathbf{S}\mathbf{C}_{11} & \mathbf{C}_{12}'\mathbf{S}\mathbf{C}_{12} \end{pmatrix}$$

The standard errors of the fixed effects are computed as:

$$var(\boldsymbol{\delta}) = \mathbf{C}_{11} + \mathbf{C}_{11}\mathbf{S}\mathbf{C}_{11}$$

The variances of the random effects are then computed as follows. We first compute the following matrix:

$$\boldsymbol{T} = [\boldsymbol{I} + \left(\boldsymbol{Z}'\boldsymbol{Z} - \boldsymbol{Z}'\boldsymbol{W}(\boldsymbol{W}'\boldsymbol{W})^{-1}\boldsymbol{W}'\boldsymbol{Z}\right)\boldsymbol{D}\frac{1}{\sigma^2_{e,\eta-1}}$$

Where $\sigma^2_{e,\eta-1}$ is the estimate of σ^2_e from the previous iteration and I is the identity matrix with dimensions equal to the model matrix for the random effects and all other matrices are as defined in the prior section.

We then compute

$$\sigma^2_{e,\eta-1} = \frac{\boldsymbol{y}'\boldsymbol{e}}{N-p} - \frac{1}{N}\left(\sum_{i=1}^{N}\left(\delta_1^2\sigma^2_{u,t-1,i} + \delta_2^2\sigma^2_{u,t-2,i}\right)\right)$$

and

$$\sigma^2_q = \frac{\boldsymbol{\theta}_q^{\eta-1\prime}\boldsymbol{\theta}_q^{\eta-1}}{m_q - tr(\boldsymbol{T}_q)}$$

Where $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{W}\widehat{\boldsymbol{\delta}} - \boldsymbol{Z}\widehat{\boldsymbol{\theta}}$, $\theta_q^{\eta-1}$ are the predictions of the random effects for the level q at iteration $\eta - 1$, m_q are the number of units (teachers or schools) at level q, and $tr(\boldsymbol{T_q})$ is the trace of the matrix $\boldsymbol{T_q}$, which is the block of the matrix T corresponding to the qth level.

Construction of Z matrix

Construction of the $\boldsymbol{Z}$ matrix determines how expectations are determined for each teacher. For this section, let *i* index students, *j* index teachers, and *k* index schools. The table below summarizes how $\boldsymbol{Z}$ is constructed to achieve the expectation of effects intended by the business rules.

Table 2. Rules for constructing the Z matrix

Rule	Summary	Example
A.1	If a student is in a single subject-relevant course with a single teacher	Teacher *j* in school *k* teaches student *i* in a single course, $\tilde{z}_{ijk} = 1$
A.2	Students enrolled in the same course in multiple periods with the same teacher treated as a single student in a single course	Teacher *j* in school *k* teaches student *i* in multiple periods of a single course, $\tilde{z}_{ijk} = 1$
A.3	Students enrolled in different courses with same teacher, the growth expectation is based on the number of courses and 100 percent attribution is made to the teacher for each course	Teacher *j* in school *k* teaches student *i* in two different courses, $\tilde{z}_{ijk} = 2$. If it is 3 different courses, $\tilde{z}_{ijk} = 3$.
A.4	Students enrolled in different courses with different teachers, the growth expectation is based on the number of courses and 100 percent attribution is made to each teacher for each course	Teachers *j* and *j'* each teach student *i* in one course (but different course codes) in school *k*, $\tilde{z}_{ijk} = 1$ and $\tilde{z}_{ij'k} = 1$
A.5	Students taking the same course under multiple teachers (e.g. coteaching) will be treated as if each teacher taught the course on their own	Ifeachers *j={1,2,…J}* teach student *i* in a single course (same course number) in school *k*, $\tilde{z}_{ijk} = 1$ for each teacher. If the student takes additional courses with the teacher, $\tilde{z}_{ijk}$ will be augmented per A.2.

Note that teachers with only a single student within a subject are eliminated from the dataset, and when two teachers teach exactly the same set of students only one is retained in data used for analysis. The same teacher value-added score is then assigned to the teacher that was dropped. Each $\tilde{z}_{ijk}$ corresponds to a unique student/teacher/school combination. These elements $\tilde{z}_{ijk}$ are normalized to sum one for each student, so that $z_{ijk} = \frac{\tilde{z}_{ijk}}{\sum_{j'}\sum_{k'}\tilde{z}_{ij'k'}}$. These elements are summed within teachers across schools to create the elements z_{ij} of the $\boldsymbol{Z_1}$ matrix, and the elements are summed within schools across teachers to create the elements z_{ik} of the $\boldsymbol{Z_2}$ matrix.

Construction of W matrix

The model matrix **W** consists of the following:

- A constant (The expected value of y when all W=0)
- Up to two prior years of achievement scores
- A missing score indicator for the second prior score
- Up to 14 Students with Disabilities (SWD) status indicators
- Gifted status
- 4 English Language Learner (ELL) status indicators (time as ELL)
- Attendance (percent of days present)
- Number of transitions between schools
- Difference from modal age in grade
- Number-of-course indicators
- Homogeneity of entering test scores in the first class in which the student is enrolled. This variable is always based on the immediate (see table defining prior grades) prior Mathematics scores for entering students Compute interquartile range based on all entering scores simultaneously
- Homogeneity of entering test scores in each additional class in which the student is enrolled, up to 6
- Missing indicator(s) for the course homogeneity covariates
- Class size of the first course in which the student is enrolled
- Class size of each additional class in which the student is enrolled, up to 6
- Missing indicator(s) for the class-size covariates.

For the Algebra EOC Model this also includes:

- Percent at modal grade in the first course in which the student is enrolled
- Percent at modal grade in each additional class in which the student is enrolled, up to 6
- Missing indicator(s) for percent-at-modal-grade covariates
- Percent gifted in the first course in which the student is enrolled
- Percent gifted in each additional class in which the student is enrolled, up to 6
- Missing indicator(s) for the percent gifted indicators
- Mean prior test score for the first course in which the student is enrolled
- Mean prior test score for each additional class in which the student is enrolled, up to 6
- Missing indicator(s) for mean prior test score covariates

Standard Errors of Measurement (SEMs) at the Highest (HOSS) and Lowest (LOSS) Observed Scale Scores

The initial model runs use no adjustment to the standard errors of measurement (SEMs) at the highest and lowest observed scale scores (HOSS/LOSS). If the model converges successfully no adjustments are needed to address the issue surrounding the observation of negative variance. If the residual variance becomes negative, the starting values of the variance components are increased. If the residual variance remains negative, the SEMs of all prior scores for some records at the extreme ends of the distribution are modified. The following rules regarding adjusting for these outliers are as follows:

1. Divide the SEMs of the highest observed scale score and the lowest observed scale score by 2, and rerun the model. If it converges, STOP.
2. If the residual variance goes negative, divide the SEMs of the highest observed scale score and the lowest observed scale score by 4, and rerun the model. If it converges, STOP.
3. If the residual variance goes negative, divide the SEMs of the highest observed scale score and the lowest observed scale score by 8, and rerun the model. If it converges, STOP.
4. If the residual variance goes negative, divide the SEMs of the highest observed scale score and the lowest observed scale score by 16, and rerun the model. If it converges, STOP.
5. If the residual variance goes negative, divide the SEMs of the 5 highest observed scale scores and the 5 lowest observed scale scores by 2, and rerun the model. If it converges, STOP.
6. If the residual variance goes negative, divide the SEMs of the 5 highest observed scale scores and the 5 lowest observed scale scores by 4, and rerun the model. If it converges, STOP.
7. If the residual variance goes negative, divide the SEMs of the 5 highest observed scale scores and the 5 lowest observed scale scores by 8, and rerun the model. If it converges, STOP.
8. If the residual variance goes negative, divide the SEMs of the 5 highest observed scale scores and the 5 lowest obtainable scale scores by 16, and rerun the model.

Section 3: Final Estimates of the Teacher Value-Added Score

For the Algebra I EOC models, a teacher's value added score is simply the empirical Bayes estimate of the teacher effect produced by the model for the one year. For the statewide, standardized assessment models in English/Language Arts and Mathematics, as described previously, the Student Growth Implementation Committee (SGIC) recommended that some of the unique school component be added back to the teacher effect.

The teacher value-added score can then be expressed as follows:

$$\xi_j^* = \xi_j + .5\xi_{k(j)}$$

Where ξ_j is the empirical Bayes estimate of the teacher effect, $\xi_{k(j)}$ is the empirical Bayes estimate of the school component and the notation *k(j)* is used to mean that teacher *j* is in school *k*. Because the revised teacher effect is a linear combination of the teacher and school effects, the final conditional variance of the teacher effect no longer applies and a new variance estimator is required. However, this is easily established using the conditional variances of the empirical Bayes estimates as the variance of the linear combination, which is denoted as follows:

$$var(\xi_j^*) = var\left(\xi_j\right) + .25var\left(\xi_{k(j)}\right) + cov(\xi_j, \xi_{k(j)})$$

Section 4: Aggregating Scores across Subjects and Years

Many teachers receive value-added scores in more than one grade or subject, and teacher value-added scores are aggregated over time. For example, a 4th grade teacher might receive value-added scores for both grade 4 Mathematics and grade 4 ELA. Similarly, a middle school Mathematics teacher might receive value-added scores for grade 7 Mathematics and grade 8 Mathematics and receive scores in each area for the past three years. Because the variability of scores can differ in each of these scenarios, it is necessary to standardize the scores and then aggregate across subject, grade and years.

These specifications for collecting results focus on the aggregation of teacher and school value-added scores and standard errors across grades, subjects, and, in the case of teachers, schools.

Teacher Files

Eight separate teacher files are created:

- 1-year aggregate
- 2-year aggregate
- 3-year aggregate
- 1-year aggregate by grade
- 2-year aggregate by grade
- 3-year aggregates by grade
- 1-year ELA file
- 1 -year Mathematics file

The aggregate and by-grade aggregate teacher files include the following:

- Teacher name and ID
- School name and ID
- District name and ID
- Number of student scores contributing to the teacher's ELA score
- The teacher's ELA VAM score and its standard error
- Number of student scores contributing to the teacher's Mathematics score
- The teacher's Mathematics VAM score and its standard error
- Number of student scores contributing to the teacher's combined score
- The teacher's combined VAM score and its standard error
- Number of unique students whose scores contributed to the teacher composite VAM score
- Binary flags for the years (ex. 2013-14, 2012-13, and 2011-12) to indicate the years a teacher's score is aggregated across.
- Grade-level files also include a grade variable.
- The ELA and Mathematics 1-year teacher files include the following:
- Teacher name and ID
- School name and ID
- District name and ID
- Subject
- Grade
- Teacher component estimate and its standard error
- School component estimate and its standard error
- Teacher VAM score and its standard error
- Number of students with scores linked to that teacher
- The number and percent of students with scores linked to that teacher meeting expectations
- Race
- Highest degree attained

- Gender
- Years of experience

Create and Standardize Teacher VAM Scores and Their Variances

Definition of Terms:

- Let *j* index teachers, *k* index schools, *g* index grades, *s* index subjects, and *t* index years.
- Let ξ_{jkgst} be the teacher component.
- Let ξ_{kgst} be the school component.
- Let $\bar{g}_{gst}$ be the average growth (difference between scores at times *t* and *t-1*) for all students in grade *g*, subject *s*, and year *t.*

Before standardizing across grades and subjects, we first combine the teacher and school components to create the teacher VAM score as follows:

$$\xi^*_{jkgst} = \xi_{jkgst} + \left(0.5 \times \xi_{kgst}\right)$$

The variance of ξ^*_{jklgst} is then computed as follows:

$$var\left(\xi^*_{jkgst}\right) = var\left(\xi_{jkgst}\right) + 0.25 * var\left(\xi_{kgst}\right) + cov\left(\xi_{jkgst}, \xi_{kgst}\right)$$

Standardize Scores

The values of ξ^*_{jkgst} and $var\left(\xi^*_{jkgst}\right)$ must be standardized before they can be aggregated because growth along the vertically aligned developmental scale is not constant from one grade level to the next. Teacher VAM scores are standardized as follows:

$$\varphi = \frac{\xi^*_{jkgst}}{\bar{g}_{gst}}$$

The variance of a ratio with two random variables is determined from the first-order Taylor series expansion:

$$var(\varphi_{jkgst}) = \left[\frac{\partial \varphi_{jkgst}}{\partial\, \xi^*_{jkgst}}, \frac{\partial \varphi_{jkgst}}{\partial \bar{g}_{gst}}\right] \begin{bmatrix} var\left(\xi^*_{jkgst}\right) & cov(\xi^*_{jkgst}, \bar{g}_{gst}) \\ cov(\xi^*_{jkgst}, \bar{g}_{gst}) & var(\bar{g}_{gst}) \end{bmatrix} \left[\frac{\partial \varphi_{jkgst}}{\partial \xi^*_{jkgst}}, \frac{\partial \varphi}{\partial \bar{g}_{gst}}\right]^T$$

The required derivatives are as follows:

$$\frac{\partial \varphi}{\partial \xi^*_{jkgst}} = \frac{1}{\bar{g}_{gst}}$$

$$\frac{\partial \varphi_{jkgst}}{\partial \bar{g}_{gst}} = -\frac{\partial \xi^*_{jkgst}}{\bar{g}^2_{gst}}$$

We also need the variance of the average growth:

$$var(\bar{g}_{gst}) = \frac{\bar{g}^2_{gst}}{2 * (n_{gst} - 1)}$$

Where n is the number of students in grade g and subject s with current-year test scores. Substituting these into the method above and expanding gives the following:

$$var(\varphi_{jkgst}) = \frac{var(\xi^*_{jkgst})\bar{g}^2_{gst} + var(\bar{g}_{gst})\xi^*_{jkgst}{}^2 - 2cov(\xi^*_{jkgst}, \bar{g}_{gst})\xi^*_{jkgst}\bar{g}_{gst}}{\bar{g}^4_{gst}}$$

Except in very small samples the covariance term will be trivial (because any teacher j's contribution to the state average is very small), and therefore can be ignored. Because the number of students in the model is large, $var(\bar{g}_{gst})$ can reasonably be approximated to equal zero. Because the covariance term and $var(\bar{g}_{gst})$ are assumed to equal zero, we are essentially treating $\bar{g}_{gst}$ as fixed, rather than random.

Create Weights

Teachers who teach multiple grades, multiple subjects, and/or over multiple years will have multiple values of φ_{jkgst}. Moreover, in most cases teachers teaching during different years, in different subjects, or in different grades will have different numbers of students or more or less variation within their different classes. These differences lead to differences in the precision of the estimates. The estimate of overall teacher value-added can be improved by accounting for these differences through a weighted average.

Define student weights as follows:

$$w_{jkgts} = \frac{n_{jkgts}}{\sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{k=1}^{K} n_{jkgts}}$$

Where n_{jkgts} is the number of students taught by teacher *j,* in school k, in grade *g*, subject *s,* and year *t.* The weights for an individual teacher will always sum to 1. The weights for an individual teacher will likely be different for different levels of aggregation due to different values in the denominators. For example, the weights used when aggregating a single year's worth of scores will be different than the weights used when aggregating several years' worth of scores.

To strike a balance between precision and simplicity we perform this calculation in two parts. First we look at aggregations within subjects, and then look at the aggregation of Mathematics and ELA. When aggregating within subjects, certain covariance terms become ignorably small, which may not be the case across subjects. The covariance terms depend on the number of students shared across estimates, and students taking a course in a subject generally do that only within one grade. Across subjects, however, teachers more often teach the same students. Most apparently, elementary teachers commonly teach all subjects to their students. Therefore, when aggregating across subjects we do not ignore the covariance term.

Aggregate Scores

The aggregated teacher estimate is calculated as follows:

$$\varphi_j = \sum_{k=1}^{K}\sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{s=1}^{S} w_{jkgts}\varphi_{jkgts}$$

Subject-, year-, grade-, subject-by-grade-, subject-by-year-, grade-by-year-, and subject-by-grade-by-year-specific scores are calculated using analogous formulas.

Compute Variance of Aggregated Scores

The variance of the estimate φ_j is a function of the weights, the variances of the component estimates, and the sampling covariance among the estimates. Sampling covariances may be non-zero when the estimates are based, at least in part, on common students. In general, teachers rarely teach the same students across grades (students are only in one grade at a time) or over time; however, many elementary teachers teach the same students across subjects. Hence, while the cross-time, cross-grade covariances may be small enough to ignore, it unlikely that the same is true for cross-subject covariances:

$$var(\varphi_j) = \sum_g \sum_t \sum_s \sum_k w_{jkgts}^2 var(\varphi_{jkgts}) + 2 \sum_g \sum_t \sum_k w_{jkgt\,math} w_{jkgt\,read}\, cov(\varphi_{jkgt\,math}, \varphi_{jkgt\,read})$$

The sampling covariance between ELA and Mathematics estimates arises because shared students induce dependence between samples. A simple and accurate approximation can capture this covariance.

First, define r_{igts} as the residual for student *i* in grade *g* at time *t* in subject *s* (actual value less the expected value less the estimated school and teacher components). Recall that the estimates have been scaled by dividing by the average growth (difference between scores at times *t* and *t-1*) for all students in grade *g*, subject *s*, and year *t.* To place the residuals on the same scale divide them by $\bar{g}_{gts}$:

$$\hat{r}_{igts} = \frac{r_{igts}}{\bar{g}_{gts}}$$

Also, define $p_{jkgt} = \frac{n_{jkgt\,common}}{n_{jkgt\,math} \times n_{jkgt\,read}}$, where $n_{jkgt\,common}$ is the number of students taught both Mathematics and ELA by teacher j during year t in grade g. This can be calculated by subtracting the number of unique students from the sum of students taught in Mathematics and ELA separately.

With these in hand, we can approximate

$cov(\varphi_{jkgt\,math}, \varphi_{jkgt\,read}) \approx p_{jkgt} cov(\hat{r}_{igt\,math}, \hat{r}_{igt\,read})$, where the final covariance term is the Pearson's correlation between the scaled Mathematics and ELA residuals.

Share of Students Meeting Expectations

For each teacher, calculate and report the share of students who meet expectations. A student meets expectations if either of the following is true:

- The student's outcome score is greater than or equal to the student's expected score. The student's expected score is calculated using the fixed effects but not the random effects ($\hat{\boldsymbol{y}} = \boldsymbol{W}\hat{\boldsymbol{\delta}}$).
- The student's outcome score is at the highest observed scale score.

School Files

Eight separate school files are created:

- 1-year aggregate
- 2-year aggregate
- 3-year aggregate
- 1-year aggregate by grade
- 2-year aggregate by grade

- 3-year aggregates by grade
- 1-year ELA file
- 1 -year Mathematics file

The aggregate and by-grade aggregate school files include the following:

- School name and ID
- District name and ID
- Number of student scores contributing to the school's ELA score
- A weighted average of the ELA VAM scores of teachers at the school
- The standard error of that average ELA score
- Number of student scores contributing to the average ELA score
- A weighted average of the Mathematics VAM scores of teachers at the school
- The standard error of that average Mathematics score
- Number of student scores contributing to the average Mathematics score
- A weighted average of the combined VAM scores of teachers at the school
- The standard error of that average combined score
- Number of student scores contributing to the average combined score
- The number of unique students with scores linked to that school
- Binary flags for the years (ex. 2013-14, 2012-13, and 2011-12) to indicate the years a teacher's score is aggregated across.
- Grade-level files also include a grade variable.

The ELA and Mathematics 1-year school files include the following:

- School name and ID
- District name and ID
- Subject
- Grade
- Number of teachers linked to the school in that subject
- The school component and its standard error
- A weighted average of the teacher components of teachers linked to that school
- The standard error of that average teacher component
- A weighted average of the VAM scores of teachers linked to that school
- The standard error of that average VAM score
- Number of students meeting expectations
- Percent of students meeting expectations
- Number of students
- VAM scores of teachers at the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentile in that district.
- Percent of students at that school qualifying for free- or reduced-price meals
- Percent of students at that school who are non-white
- A binary variable indicating whether the school is a Title I school.

Weighted averages are student-weighted averages. These averages and their standard errors are calculated using formulae analogous to those used to calculate aggregate teacher scores and their standard errors.

$$\bar{\varphi}_k = \sum_{j=1}^{J} w_j \varphi_j$$

$$w_{j'} = \frac{n_{j'}}{\sum_{j=1}^{J} n_j}$$

$$var(\bar{\varphi}) = \sum_{j=1}^{J} w_j^2 var(\varphi_j)$$

District Files

Eight separate district files are created:

- 1-year aggregate
- 2-year aggregate
- 3-year aggregate
- 1-year aggregate by grade
- 2-year aggregate by grade
- 3-year aggregates by grade
- 1-year ELA file
- 1 -year Mathematics file

The aggregate and by-grade aggregate district files include the following:

- District name and ID
- Number of student scores contributing to the district's ELA score
- A weighted average of the ELA VAM scores of teachers at the district
- The standard error of that average ELA score
- Number of student scores contributing to the average ELA score
- A weighted average of the Mathematics VAM scores of teachers at the district
- The standard error of that average Mathematics score
- Number of student scores contributing to the average Mathematics score
- A weighted average of the combined VAM scores of teachers at the district
- The standard error of that average combined score
- Number of student scores contributing to the average combined score
- The number of unique students with scores linked to that district
- Binary flags for the years (ex. 2013-14, 2012-13, and 2011-12) to indicate the years a teacher's score is aggregated across.
- Grade-level files also include a grade variable.

The ELA and Mathematics 1-year district files include the following:

- District name and ID

- Subject
- Grade
- Number of schools in the district
- A weighted average of the school components linked to that district
- The standard error of that weighted average
- A weighted average of the VAM scores of teachers linked to that district
- The standard error of that average VAM score
- Number of students meeting expectations
- Percent of students meeting expectations
- Number of students
- VAM scores of teachers at the 5^{th}, 10^{th}, 25^{th}, 50^{th}, 75^{th}, 90^{th}, and 95^{th} percentile in that district.

Appendix A – Variables in the Reference Files

The Reference Files are compiled as follows. Table A-1 lists the variables to be retained from the original data files and the data files from which they are retained. New variables that need to be created from these original variables are listed in Table A-2.

Table A-1. Variables included from the original data file

The total number of ClassIDs for a student will dictate the iteration of the subscript ***i*** for each of the subscripted variables in the table. Note that 'yy' represents the year for which data are being reported. For example, using three years of data, we would expect (yy=14, yy=13, yy=12). If a given student has N unique ClassIDs in which he/she is enrolled, the subscript i would iterate (i=1 to N) in the table.

Original variable name	**Variable name in reference file**	**Source file**	**Description of variable**
Student_Unique_ID	SSID	Multiple	Unique Student Identifier
Year	_yy_Year	Multiple	Year
Survey	_yy_NewSurvey	Multiple	1 if Full Year, 2 if only Survey 2, 3 if Only Survey 3
District_ID	_yy_District_ID_i	Multiple	District ID
School_ID	_yy_School_ID_i	Multiple	School ID
FirstName	_yy_FirstName	Student Demographic	Student first name
LastName	_yy_LastName	Student Demographic	Student last name
SBday	_yy_S_Bday	Student Demographic	Calendar day of birth
S_Bmonth	_yy_S_Bmonth	Student Demographic	Month of birth
S_Byear	_yy_S_Byear	Student Demographic	Year of birth
S_EconDisadvantaged	_yy_S_EconDisadvantaged	Student Demographic	Binary indicator {0, 1}

S_Gender	_yy_S_Gender	Student Demographic	Categorical
S_Race	_yy_S_Race	Student Demographic	Categorical
S_LEP	_yy_S_LEP	Student Demographic	Limited English Proficiency
Entry_Date	_yy_ELL_Entry_Date	Student ELL	ELL Entry date
EXCEPTIONALITYx	_yy_SWD	Student Exceptionality	Binary variable indicating the student has at least one of the 14 SWD exceptionalities listed below. The individual _yy_SWDx variables are also binary indicators. Note that a student with a gifted exceptionality is only SWD if the student has one of the other exceptionality codes as well.
Present_Days_NBR	_yy_Present_Days_NBR	Student Attendance	Number of days the student attended school
Absent_Days_NBR	_yy_Absent_Days_NBR	Student Attendance	Number of days the student was absent from school
Course_Number	_yy_Course_Number_i	Student Course Linkage, Teacher Course Linkage	Course Number
Period	_yy_Period_i	Student Course Linkage, Teacher Course Linkage,	Course Period
Teacher_ID	_yy_Teacher_ID_i	Teacher Course Linkage	Teacher ID
ScaleScore	_yy_ScaleScore	Assessment	Scale score for year yy
ScaleScore_SEM	_yy_Scale_Score_SEM	Assessment	Standard Error of Measure (SEM) of that scale score for year yy
Grade	_yy_TestedGrade	Assessment	Tested grade of that scale score for year yy
ScaleScore	_yy-1_ScaleScore	Assessment	Scale score for year yy-1

ScaleScore_SEM	_yy-1_Scale_Score_SEM	Assessment	SEM of that scale score for year yy-1
Grade	_yy-1_TestedGrade	Assessment	Tested grade of that scale score for year yy-1
ScaleScore	_yy-2_ScaleScore	Assessment	Scale score for year yy-2
ScaleScore_SEM	_yy-2_Scale_Score_SEM	Assessment	SEM of that scale score for year yy-2
Grade	_yy-2_TestedGrade	Assessment	Tested grade of that scale score for year yy-2

Table A-2. Variables computed in the reference files

Variable name in reference file	**Description**
Subject	Mathematics or ELA
_yy_ELL_LY	Student is an English language learner. _yy_ELL_LY = 1 if and only if S_LEP = LY, and zero otherwise
_yy_ELL_LY_1	For the current year, using the S_LEP variable, create a new variable, ELL_LY_1, which is 1 if and only if S_LEP=LY and the testing date minus the entry date is less than two years. Otherwise, ELL_LY_1 = 0.
_yy_ELL_LY _2	For the current year, using the S_LEP variable, create a new variable, ELL_LY_2, which is 1 if and only if S_LEP=LY and the testing date minus the entry date is at least two years but less than four years. Otherwise, ELL_LY_2 = 0.
_yy_ELL_LY _3	For the current year, using the S_LEP variable, create a new variable, ELL_LY_3, which is 1 if and only if S_LEP=LY and the testing date minus the entry date is at least four years but less than six years. Otherwise, ELL_LY_3 = 0.
_yy_ELL_LY _4	For the current year, using the S_LEP variable, create a new variable, ELL_LY_4, which is 1 if and only if S_LEP=LY and the testing date minus the entry date is equal to or greater than six years. Otherwise, ELL_LY_4 = 0.
_yy_S_Gifted	Set value to 1 if and only if EXCEPTIONALITY = L (indicating Gifted). Otherwise, set value to 0.

_yy_SWD1	Set value to 1 if and only if EXCEPTIONALITY = A (indicating Intellectual Disability) (Collapsed into Code W in 2008-09). Otherwise, set value to 0
_yy_SWD2	Set value to 1 if and only if EXCEPTIONALITY = B (indicating Intellectual Disability)(Collapsed into Code W in 2008-09). Otherwise, set value to 0.
_yy_SWD3	Set value to 1 if and only if EXCEPTIONALITY = G (indicating Language Impaired). Otherwise, set value to 0.
_yy_SWD4	Set value to 1 if and only if EXCEPTIONALITY = H (indicating Deaf or Hard of Hearing). Otherwise, set value to 0.
_yy_SWD5	Set value to 1 if and only if EXCEPTIONALITY = I (indicating Visually Impaired). Otherwise, set value to 0.
_yy_SWD6	Set value to 1 if and only if EXCEPTIONALITY = J (indicating Emotional/Behavioral Disability. Otherwise, set value to 0.
_yy_SWD7	Set value to 1 if and only if EXCEPTIONALITY = K (indicating Specific Learning Disability). Otherwise, set value to 0.
_yy_SWD8	Set value to 1 if and only if EXCEPTIONALITY = N (indicating Intellectual Disability) (Collapsed into Code W in 2008-09). Otherwise, set value to 0.
_yy_SWD9	Set value to 1 if and only if EXCEPTIONALITY = O (indicating Dual-Sensory Impaired. Otherwise, set value to 0.
_yy_SWD10	Set value to 1 if and only if EXCEPTIONALITY = P (indicating Autism Spectrum Disorder. Otherwise, set value to 0.
_yy_SWD11	Set value to 1 if and only if EXCEPTIONALITY = Q (indicating Emotional / Behavioral Disability (Collapsed into Code J in 2007-08). Otherwise, set value to 0.
_yy_SWD12	Set value to 1 if and only if EXCEPTIONALITY = S (indicating Traumatic Brain Injured. Otherwise, set value to 0.
_yy_SWD13	Set value to 1 if and only if EXCEPTIONALITY = V (indicating Other Health Impaired. Otherwise, set value to 0.
_yy_SWD14	Set value to 1 if and only if EXCEPTIONALITY = W (indicating Intellectual Disability. Otherwise, set value to 0.
_yy_Bdate	Birthdate (SAS Date value)
_yy_Age	Student age is calculated as the age in years as of September 1 of the academic year.
_yy_ModalAge	Modal age in grade, where age is calculated as the age in years as of September 1 of the academic year.

_yy_DeltaAge	Difference from modal age in grade: yy_Age – yy_ModalAge
_yy_Present_Days_Prop	Across all schools, sum the number of days in attendance. If the sum is greater than 180 days, set equal to 180 days. Across all schools, sum the number of days in attendance and the number of days absent to create the total number of days enrolled. If the sum is greater than 180 days, set equal to 180 days. Divide by the total days in attendance by the total days enrolled to obtain the share of days in attendance.
_yy_Teacher_Effect_i	Attribution of the teacher effect. The weight for a teacher divided by the sum of the weights of all teachers for the specific student: $\frac{z_{i'j}}{\sum_{j=1}^{J} z_{i'j}}$. The weight of 1 would be assigned to a teacher if they were the only teacher the student was assigned; while a weight of 0.5 would be assigned if the student had two different teachers.
_yy_Num_Trans	Sort the attendance file by year, SSID, and entry date. If the student has only one record within the current school year, he/she has 0 transitions. For each change of school, within the year, count one transition. If a student has two entry dates for the same school, count as one transition only if the second entry date is more than 21 days after the previous withdrawal date.
_yy_NumberCourses	Total number of subject relevant courses in which the student is enrolled.
Courses_x_or_more	A vector of binary variables indicating the number of courses in which the student is enrolled. 2 or more, 3 or more, 4 or more, 5 or more.
_yy_Class_Size_i	The count of students who are enrolled in the same course with the same teacher during the same period at the same school in the same district.
_yy_Homogeneity_i	The interquartile range of year yy-1 test scores among all students who are enrolled in the same course with the same teacher during the same period at the same school in the same district.
_yy_Mean_Prior_i	Mean prior assessment score among all students in class.
_yy_Pct_Gifted_i	Percent of students in class who are gifted.
_yy_Pct_AtModalGrade_i	Percent of students in class who are at the modal grade.