Nonparametric Statistical Methods

SECOND EDITION

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bivariate distribution for which $\tau = .4$. Since $z_\alpha = z_{.01} = 2.326$ and $z_\beta = z_{.10} = 1.282$, we find that the approximate required sample size for the alternative $\tau = .4$ is

$$n \doteq \frac{4(2.326 + 1.282)^2}{9(.4)^2} = 36.2.$$

To be conservative, we would take $n = 37$.

14. *Trend Test*. If we take $X_i = i, i = 1, \ldots, n$ and consider

$$K = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Q((i, Y_i), (j, Y_j))$$
$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c(Y_j - Y_i),$$

where

$$c(a) = \begin{cases} 1 & \text{if } a > 0, \\ 0 & \text{if } a = 0, \\ -1 & \text{if } a < 0, \end{cases}$$

then K can be used as a test for a time trend in the univariate random sample $Y_1, \ldots, Y_n$. This use of K to test for a time trend was suggested by Mann (1945).

15. *Other Uses for the K Statistic*. Wilcoxon's rank sum test (Section 4.1) and Jonckheere-Terpstra's test (Section 6.2) can be viewed as tests based on K (8.6) (or, equivalently, $\bar{\tau}$ (8.34)). For this interpretation see Jonckheere (1954a) and Kendall (1962, Sections 3.12 and 13.9). Also, Wolfe (1977) has used the K statistic to compare the correlation between variables X_2 and X_1 with that between the variables X_3 and X_1, when both X_2 and X_3 are potential predictors for X_1.

16. *Consistency of the K Test*. Under the assumption that $(X_1, Y_1), \ldots, (X_n, Y_n)$ is a random sample from a continuous bivariate population with joint distribution function $F_{X,Y}(x, y)$, the consistency of the tests based on K depends on the parameter τ (8.2). The test procedures defined by (8.8), (8.9), and (8.10) are consistent against the class of alternatives corresponding to $\tau >, <$, and $\neq 0$, respectively.

17. *Multivariate Concordance*. Joe (1990) has generalized Kendall's measure of association τ from the bivariate case where τ measures the strength of association between two variables X, Y to the multivariate case where $\mathbf{X} = (X_1, \ldots, X_m)$ is an m-dimensional random variable and one is interested in a measure of the strength of the association between the components $X_1, \ldots, X_m$ of $\mathbf{X}$. Let F denote the joint distribution function of $\mathbf{X}$,

$$F(x_1, \ldots, x_m) = P(X_1 \leq x_1 \text{ and } X_2 \leq x_2 \text{ and } \ldots \text{ and } X_m \leq x_m)$$

and denote the marginal distribution functions as $F_j(x_j) = P(X_j \leq x_j), j = 1, \ldots, m$. The null hypothesis of mutual independence of $X_1, \ldots, X_m$ is

$$H_0 : F(x_1, \ldots, x_m) = \prod_{j=1}^{m} F_j(x_j), \quad \text{for all } (x_1, \ldots, x_m).$$

That is, the joint distribution is equal to the product of the marginals.

TABLE A.30. Upper-Tail Probabilities for the Null Distribution of the Kendall K Statistic: $n = 4(1)40$

For a given n, the entry in the table for the point x is $P_0\{K \geq x\}$. Under these conditions, if x is such that $P_0\{K \geq x\} = \alpha$, then $k_\alpha = x$. For certain n, the entries are terminated at x_n, where x_n is the smallest possible value of x such that $P_0\{K \geq x\}$ is zero to three decimal places. (For $n = 4(4)40$ or $n = 5(4)37$, all even integers between $-n(n-1)/2$ and $n(n-1)/2$ have positive probability and for $n = 6(4)38$ or $n = 7(4)39$ all odd integers between $-n(n-1)/2$ and $n(n-1)/2$ have positive probability.)

	n								
x	4	5	8	9	12	13	16	17	20
0	.625	.592	.548	.540	.527	.524	.518	.516	.513
2	.375	.408	.452	.460	.473	.476	.482	.484	.487
4	.167	.242	.360	.381	.420	.429	.447	.452	.462
6	.042	.117	.274	.306	.369	.383	.412	.420	.436
8		.042	.199	.238	.319	.338	.378	.388	.411
10		.008	.138	.179	.273	.295	.345	.358	.387
12			.089	.130	.230	.255	.313	.328	.362
14			.054	.090	.190	.218	.282	.299	.339
16			.031	.060	.155	.184	.253	.271	.315
18			.016	.038	.125	.153	.225	.245	.293
20			.007	.022	.098	.126	.199	.220	.271
22			.002	.012	.076	.102	.175	.196	.250
24			.001	.006	.058	.082	.153	.174	.230
26			.000	.003	.043	.064	.133	.154	.211
28				.001	.031	.050	.114	.135	.193
30				.000	.022	.038	.097	.118	.176
32					.016	.029	.083	.102	.159
34					.010	.021	.070	.088	.144
36					.007	.015	.058	.076	.130
38					.004	.011	.048	.064	.117
40					.003	.007	.039	.054	.104
42					.002	.005	.032	.046	.093
44					.001	.003	.026	.038	.082
46					.000	.002	.021	.032	.073
48						.001	.016	.026	.064
50						.001	.013	.021	.056
52						.000	.010	.017	.049
54							.008	.014	.043
56							.006	.011	.037
58							.004	.009	.032
60							.003	.007	.027
62							.002	.005	.023
64							.002	.004	.020
66							.001	.003	.017
68							.001	.002	.014
70							.001	.002	.012
72							.000	.001	.010
74								.001	.008
76								.001	.007
78								.000	.006
80									.005
82									.004
84									.003
86									.002
88									.002